PANAMÁ 2020

OMCC

XXII Olimpiada Matemática de Centroamérica y El Caribe

Panama City, Panama

Day 2
October 28th, 2020

Problem 4

Consider a triangle ABC with $BC > AC$. The circle with center C and radius AC intersects the segment BC in D. Let I be the incenter of triangle ABC and Γ be the circle that passes through I and is tangent to the line CA at A. The line AB and Γ intersect at a point F with $F \neq A$. Prove that $BF = BD$.

Problem 5

Let $P(x)$ be a polynomial with real non-negative coefficients. Let k be a positive integer and $x_1, x_2, \ldots, x_k$ positive real numbers such that $x_1 x_2 \cdots x_k = 1$. Prove that

$$P(x_1) + P(x_2) + \cdots + P(x_k) \geq kP(1).$$

Problem 6

A positive integer N is said to be *interoceanic* if its prime factorization

$$N = p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

satisfies that

$$x_1 + x_2 + \cdots + x_k = p_1 + p_2 + \cdots + p_k.$$

Find all interoceanic numbers less than 2020.

Time: 4 hours and 30 minutes
Each problem is worth 7 points