OMCC

PANAMÁ 2020

XXII Olimpiada Matemática de Centroamérica y El Caribe

Panama City, Panama

Day 1
October 27th, 2020

Problem 1

A four-digit positive integer is called *virtual* if it has the form $\overline{abab}$, where a and b are digits and $a \neq 0$. For example 2020, 2121 and 2222 are virtual numbers, while 2002 and 0202 are not. Find all virtual numbers of the form $n^2 + 1$, for some positive integer n.

Problem 2

Suppose you have identical coins distributed in several piles with one or more coins in each pile. An action consists of taking two piles, which have an even total of coins among them, and redistribute their coins in two piles so that they end up with the same number of coins.

A distribution is *levelable* if it is possible, by means of 0 or more operations, to end up with all the piles having the same number of coins.

Determine all positive integers n such that, for all positive integers k, any distribution of nk coins in n piles is levelable.

Problem 3

Let $\mathbb{Z}$ be the set of integers. Find all the functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying the following property:

If a, b and c are integers such that $a + b + c = 0$, then

$$f(a) + f(b) + f(c) = a^2 + b^2 + c^2.$$

Time: 4 hours and 30 minutes
Each problem is worth 7 points